Determinants and Cramer's rule

-Every square matrix can be associated with a real number called its **determinant.**

-Historically, the use of determinants arose from special number patterns that occur when systems of linear equations are solved.

Determinant of a 2 x 2 matrix

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

$a_2 b_1$

$a_1 b_2$

The <u>determinant</u> is defined as:

bottom - top

$$\det(A) = |A| = a_1 b_2 - a_2 b_1$$

Find the determinant:

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \quad \begin{matrix} -3 \\ 4 \end{matrix}$$

$4 - (-3) = 7$

Determinant of a 3x3

*Not the way the book teaches it!

$+ \quad + \quad = \text{top}$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

bottom − top =

$+ \quad + \quad = \text{bottom}$

Find the determinant of:

$$\begin{array}{ccc} & & & & & -4 & 0 & 6 & = 2 \end{array}$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{matrix} 0 & 2 \\ 3 & -1 \\ 4 & 0 \end{matrix}$$

$$0 \quad 16 \quad 0 = 16$$

$16 - 2 = 14$

Let's look at:

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

solve for x

$$3x + 2y = 7$$
$$-5x + 4y = -9$$

Cramer's Rule

answers

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

coeff.

answer

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

coeff.

Use Cramer's Rule to solve:

$$4x - 2y = 10$$
$$3x - 5y = 11$$

$$x = \frac{\begin{vmatrix} 10 & -2 \\ 11 & -5 \end{vmatrix}^{-22}_{-50}}{\begin{vmatrix} 4 & -2 \\ 3 & -5 \end{vmatrix}^{-6}_{-20}} = \frac{-28}{-14} = 2$$

$$y = \frac{\begin{vmatrix} 4 & 10 \\ 3 & 11 \end{vmatrix}^{30}_{44}}{-14} \quad \frac{14}{-14}$$

$$y = -1$$

$$-2x + 5y = 9$$
$$7x - 2y = 17$$

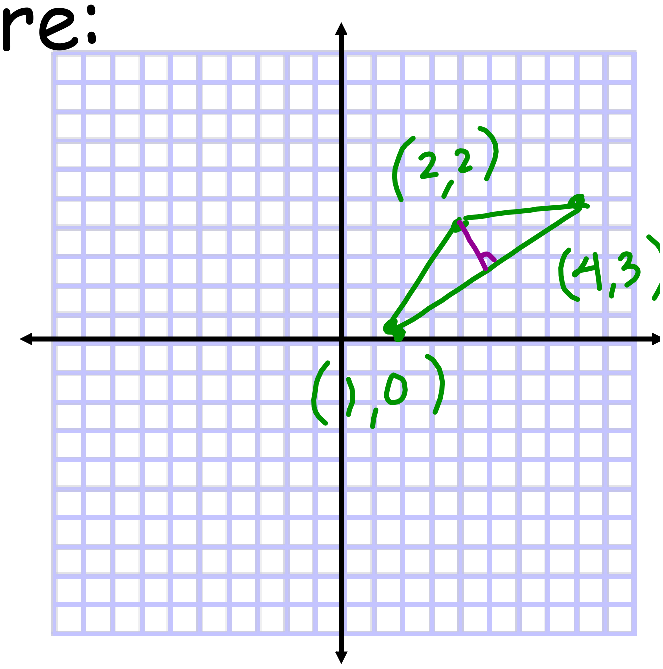
$x =$ ______

$y =$ ______

$$-x + y - z = -14$$
$$2x - y + z = 21$$
$$3x \quad 2y + z = 19$$

Find the area of a triangle whose vertices are:
(1,0), (2,2), (4,3)

$\frac{1}{2}bh$



Area of a Triangle:

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is:

$$\text{Area} = \pm\frac{1}{2}\underbrace{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}_{bh}$$

Find the area of a triangle whose vertices are:

(1,0), (2,2), (4,3)

$8 \quad 3 \quad 0 = 11$

$$= \pm \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 2 & 2 \\ 4 & 3 \end{matrix} \qquad -3$$

$2 \quad 0 \quad 6 = 8$

$$A = -\frac{1}{2}(-3)$$

$$A = 3/2$$